The relaxation time is also related to mean free path λ by the relation

$$\lambda = | v_F | 2\tau,$$

where v_F is speed of the electrons at the top of Fermi distribution,

i.e. $$\tau = \frac{\lambda}{2 | v_F |} \qquad ...(6)$$

Now $$\frac{1}{\tau} = 2 | v_F | \left(\frac{1}{\lambda_T} + \frac{1}{\lambda_d} + \frac{1}{\lambda_I} \right) = \frac{1}{\tau_T} + \frac{1}{\tau_d} + \frac{1}{\tau_I} \qquad ...(7)$$

The resistivity is defined as $\rho = 1/\sigma$, therefore

$$\rho = \frac{m^*}{Ne^2\tau}$$

$$= \frac{m^*}{Ne^2\tau_T} + \frac{m^*}{Ne^2\tau_d} + \frac{m^*}{Ne^2\tau_I} \qquad ...(8)$$

If the first term on right hand side is defined as ρ_T, second as ρ_d and the third as ρ_I, then

$$\rho = \rho_T + \rho_d + \rho_I. \qquad ...(9)$$

This shows that resistivities are additive which is Mathiessen's rule.

8·9. THE HALL EFFECT :

The conductivity measurements are not sufficient for the determination of the number of conducting charge N and their mobility μ. Moreover these measurements do not give any information about the sign of the prominant charge carrier. The Hall effect supplies the information of the sign of charge carrier.

(When a magnetic field is applied perpendicular to a conductor carrying current, a voltage is developed across the specimen in the direction perpendicular to both the current and magnetic field. This phenomenon is known as Hall effect.)

Consider that an external electric field is applied along the axis of a specimen, then the electrons will drift in opposite direction. Again let a magnetic field be applied perpendicular to the axis of the specimen then the electrons will tend to be deflected to one side. Of course, the electrons will not drift into space but a surface charge is developed. The surface charge then gives rise to a transverse electric field which causes a compensating drift such that the carriers remain in the specimen. The effect is known as Hall effect. The hall effect is thus observed when a magnetic field is applied at right angle to a conductor carrying a current.

Consider a slab of material subjected to an external electric field $\mathcal{E}_x$ along the x-direction and a magnetic field Hz along the z-direction as shown in figure 4. Due to the electric field a current density I_x will flow in the direction of $\mathcal{E}_x$. Let us consider

the case in which the current is carried by electrons of charge $-e$. Under the influence of the magnetic field, the electron will be subjected to a Lorentz force such that the upper surface collects a positive charge while the lower surface a negative charge. The accumulation of charge on the surface of the specimen continues until the force on moving charges due to the electric field associated with it is large enough to cancel the force exerted by the magnetic field. Ultimately a stationary state is reached when the current along y-axis vanishes, a field $\mathcal{E}_y$ is set up. If the charge carriers are holes then the case will be reversed, *i.e.* the upper surface would become negative while the lower surface as positive. Thus by measuring the Hall voltage in y-direction, the information about the sign of charge carriers may be obtained. In this way, the measurement of Hall voltage gives the information about the charge carriers.

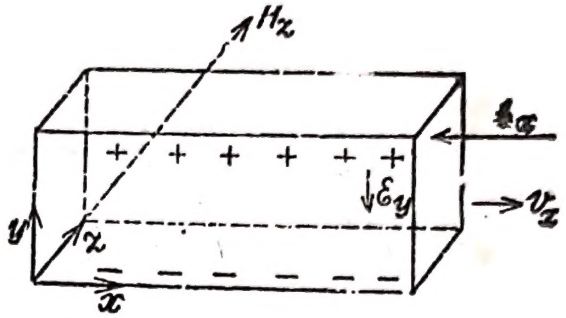

Fig. 4. The geometry ofthe electric and magnetic fields for a simple Hall effect calculation

Hall voltage and Hall coefficient :

(*First treatment*) :

The electric force on the electron having a charge $-e$ is $-e\vec{\mathcal{E}}$ and the force due to magnetic field $\mathbf{H} = -e/c\,(\mathbf{v} \times \mathbf{H})$. Due to the combined effect of electric and magnetic fields, the total force is given by

$$\mathbf{F} = -e\vec{\mathcal{E}} - \frac{e}{c}(\mathbf{v} \times \mathbf{H}). \qquad ...(1)$$

In our case

$$F_y = -e\mathcal{E}_y + \frac{e}{c} v_x H_z. \qquad ...(2)$$

In this steady state $F_y = 0$, hence

$$0 = -e\mathcal{E}_y + \frac{e}{c} v_x H_z$$

or

$$\mathcal{E}_y = \frac{1}{c} v_x H_z. \qquad ...(3)$$

This $\mathcal{E}_y$ is called Hall voltage or Hall field.

Now the current density is given by

$$I_x = N(-e)\,v_x$$

or

$$v_x = -I_x/Ne. \qquad ...(4)$$

From equation (4) substituting the value of v_x in equation (3), we have

$$\mathcal{E}_y = -\frac{1}{c}\frac{I_x}{Ne}H_z$$

or
$$\frac{\mathcal{E}_y}{I_x H_z} = \frac{-1}{cNe} = R_{Hall}$$

$$\therefore \quad R_{Hall} = -\frac{1}{cNe} \qquad \ldots(5)$$

R_{Hall} is known as Hall coefficient. The expression (5) for Hall coefficient is in e.m.u. and hence the value of Hall coefficient in e.s.u. is given by

$$R_{Hall} = -\frac{1}{Ne}. \qquad \ldots(6)$$

The expression (5) or (6) shows that the sign of Hall constant is the same as the sign of carrier. Had our calculation been based upon holes, the sign of charge entering the Lorentz force would have been positive and the corresponding Hall-constant positive.

(*Second treatment*) :

For a uniform steady state system, the Boltzmann equation can be written

$$\frac{\partial f}{\partial \mathbf{p}}\cdot\mathbf{F} = -\frac{f-f_0}{\tau}, \qquad \ldots(7)$$

because the derivative of distribution function with position vanishes in steady state system.

The force **F** includes both electric and magnetic forces. Replacing **F** by Lorentz force for electron, we have

$$\frac{\partial f_0}{\partial E}\,\mathbf{v}.\left(-e\,\vec{\mathcal{E}}-\frac{e}{c}(\mathbf{v}\times\mathbf{H})\right)+\frac{\partial f_1}{\partial \mathbf{p}}.\left(-e\,\vec{\mathcal{E}}-\frac{e}{c}(\mathbf{v}\times\mathbf{H})\right)=-\frac{f_1}{\tau}\ldots.(8)$$

Here we have divided $\partial f/\partial \mathbf{p}$ in zero order and first order contribution in order to take advantage of the dependence on energy only of the equilibrium distribution function. As $\mathbf{v}.(\mathbf{v}\times\mathbf{F})$ is zero, hence the magnetic field term in first bracket vanishes and the term$=e\,\vec{\mathcal{E}}$ in the second bracket is droped as it is of second order, hence

$$-e\frac{\partial f_0}{\partial E}\,\mathbf{v}.\vec{\mathcal{E}}-\frac{e}{c}\frac{\partial f_0}{\partial \mathbf{p}}(\mathbf{v}\times\mathbf{H})=-\frac{f_1}{\tau}. \qquad \ldots(9)$$

Roughly speaking, the effect of magnetic field is to rotate the distribution function hence we try to form a distribution function on analogy of conductivity calculation but with the difference that the electric field is replaced a general vector **G**, which will be determined, *i.e.*

$$f_1 = e\tau\frac{\partial f_0}{\partial E}\,\mathbf{v}.\mathbf{G} \qquad \ldots(10)$$

We now substitute this trial solution in Boltzmann equation and get

$$-e\frac{\partial f_0}{\partial E}\mathbf{v}\cdot\vec{\mathcal{E}}-\frac{e}{c}\left[e\tau\left\{\frac{\mathbf{G}}{m}\frac{\partial f_0}{\partial E}+\frac{\partial^2 f_0}{\partial E^2}(\mathbf{v}.\mathbf{G})\cdot\mathbf{v}\right\}\right](\mathbf{v}\times\mathbf{H})$$

$$=-e\frac{\partial f_0}{\partial E}(\mathbf{v}\cdot\mathbf{G})$$

or $$-e\frac{\partial f_0}{\partial E}\mathbf{v}\cdot\vec{\mathcal{E}}-\frac{e}{c}\left[e.\left\{\frac{\mathbf{G}}{m}\frac{\partial f_0}{\partial E}+0\right\}\right](\mathbf{v}\times\mathbf{H})=-\frac{e\partial f_0}{\partial E}(\mathbf{v}\cdot\mathbf{G})$$

or $$\mathbf{v}\cdot\vec{\mathcal{E}}+\frac{e\tau}{mc}\mathbf{G}\cdot(\mathbf{v}\times\mathbf{H})=+(\mathbf{v}\cdot\mathbf{G})$$

or $$\mathbf{v}\left[\vec{\mathcal{E}}+\frac{e\tau}{mc}(\mathbf{H}\times\mathbf{G})-\mathbf{G}\right]=0.$$

Here we have used $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})$

$$\therefore\quad \vec{\mathcal{E}}=\mathbf{G}-\frac{e\tau}{mc}(\mathbf{H}\times\mathbf{G}) \qquad ...(11)$$

The current associated with first order trial distribution function may be obtained in the same way as in case of electrical conductivity and the result is of the form $I=\sigma\mathbf{G}$.

$$\therefore\quad \vec{\mathcal{E}}=\frac{\mathbf{I}}{\sigma}-\frac{e\tau}{mc\sigma}(\mathbf{H}\times\mathbf{I}). \qquad ...(12)$$

In the absence of magnetic field the second term is zero and the result is the same as we have obtained previously.

The second term is a component of electric field which is transverse both to the applied magnetic field and to the current. The proportionality constant is called the Hall constant and is magnitude is given by

$$R_{Hall}=\frac{-e\tau}{mc\sigma}=-\frac{1}{Nec} \qquad ...(13)$$

Hall constants and Hall mobilities for some metals at room temperature are given below :

HALL CONSTANTS AND HALL MOBILITIES AT ROOM TEMPERATURE

Metal	$R_{Hall}\times 10^{10}\ \frac{\text{Volt } m^3}{\text{Amp. Weber}}$	$\mu,\ \frac{m^2}{\text{Volt sec.}}$
Ag	−0·84	0·0056
Al	−0·30	0·0012
Au	−0·72	0·0030
Cu	−0·55	0·0032
Li	−1·70	0·0018
Na	−2·50	0·0053
Zn	+0·3	0·0060
Cd	+0·7	0·0080

From eq. (13) it is observed that Hall coefficient R_{Hall} has the same sign as the carriers of current (here electrons). So if the charge carriers are electrons, R_{Hall} should always be negative. Experimentally (see table page 281), however, it has been observed that R_{Hall} is positive for many metallic and semiconducting substances (p-type). The occurrence of positive Hall coefficient indicates that the charge carriers should be positive and not negative. The free electron theory does not provide any reasonable way of the presence of positive charge carriers. This had been a problem for the scientists until an explanation was provided by band theory. According to the band theory, the vacant states near the top of an otherwise filled energy band behave like a set of positive charges. States of this character give rise to positive Hall coefficients.

Mobility and Hall angle : The *mobility* (μ) is defined as the velocity acquired by the current carrying particles per unit electric field, *i.e.*.

$$\mu=\frac{v_x}{\mathcal{E}_x}$$

$$v_x=\mu\, \mathcal{E}_x. \quad \ldots(14)$$

Substituting this value in equation (3)

$$\mathcal{E}_y=\frac{1}{c}\mu\, \mathcal{E}_x\, H_z. \quad \ldots(15)$$

Comparing equation (15) and (5), we get

$$R_{Hall}.I_x.H_z=\frac{1}{c}\,\mu\mathcal{E}_x\, H_z$$

or

$$\mu=\frac{R_{Hall}.\ I_x}{\mathcal{E}_x}.\ c$$

$$\mu=R_{Hall}.\sigma c \left(\because\ \frac{I_x}{\mathcal{E}_x}=\sigma\right) \quad \ldots(16)$$

In e.m.u.

$$\mu=R_{Hall}.\sigma$$

$$=\frac{\mathcal{E}_y\sigma}{I_x.H_z}$$

$$=\frac{\mathcal{E}_y\sigma.}{\sigma\mathcal{E}_x H_z}$$

$$=\phi\,\frac{1}{H_z} \quad \text{where } \phi=\frac{\mathcal{E}_y}{\mathcal{E}_x} \text{ is called Hall angle}$$

$$\therefore \quad \phi=\mu.H_z. \quad \ldots(17)$$

Importance of Hall effect : The measurement of the Hall effect given the following important quantities :

(1) The sign of the current carrying charges is determined.

(2) From the magnitude of Hall coefficient, the number of charge carriers per unit volume can be calculated.

(3) The mobility is measured directly.

(4) It can be used to decide whether a material is metal, semiconductor or insulator.

Here one thing should be remembered that not all the metals have a negative Hall constant but some metals have a positive Hall constant (*i.e.*, charge carries are holes) and if both holes and electrons contribute to conductivity then R_{Hall} can be positive or negative depending upon the relative densities and mobilities of the carriers.

Experimental determination of Hall coefficient : In the experimental determination of Hall coefficient a thin metallic strip of several m.m. wide and several cm. long in the x-direction is placed in a magnetic field H in the z-direction as shown in figure 5. A suitable current is allowed to flow in the specimen along the x-direction which can be subjected by variable rheostat. Two potential lead are placed between the points A and B which are connected to a sensitive calibrated potentiometer. Thus the Hall voltage is measured with the help of calibrated potentiometer. It ranges from 0·1 to tens of micro volts in typical cases so that stray potentials due to thermal effects create difficulty in these measurement. Measuring the Hall voltage, Hall coefficient can be calculated $\mathcal{E}_y$ which is given by

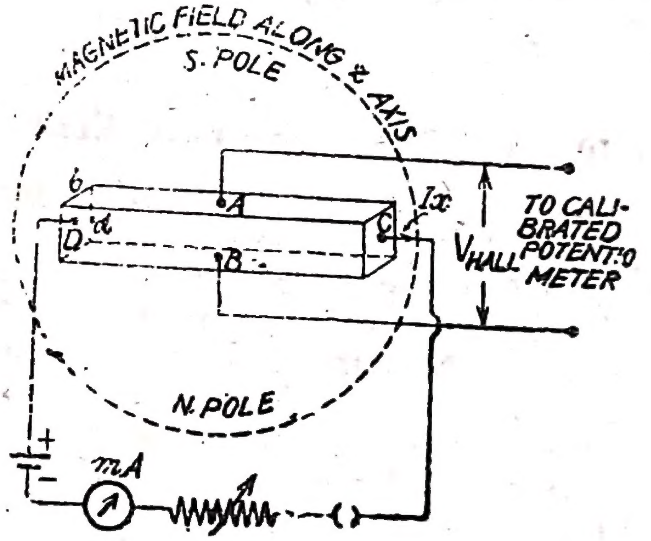

Fig. 5. Experimental measarement of Hall voltage.

$$\mathcal{E}_y = -\frac{1}{c}\frac{I_x}{Ne}H_z. \qquad \ldots(18)$$

The value of I_x can be expressed in terms of the current i, *i.e.*,

$$I_x = \frac{i}{b \times d} \qquad \ldots(19)$$

where b is the breadth of the specimen and d is the thickness.

Substituting (19) in equation (18)

$$\mathcal{E}_y = -\frac{1}{c}\frac{iH_z}{Nebd}$$

or
$$V_{Hall}=\mathcal{E}_y\, d=-\frac{1}{c}\frac{H_z i}{Neb}$$
$$=R_{Hall}\cdot\frac{iH_z}{b}$$

or
$$R_{Hall}=\frac{b\, V_{Hall}}{iH_z}. \qquad \ldots(20)$$

In this way, Hall coefficient can be calculated.

Sources of error :

(1) Due to the temperature gradient, errors are introduced. The errors are eliminated by reversing the current and taking another pair of readings with magnetic field normal and reversed.

(2) Even in the absence of magnetic field, a potential difference due to imperfect alignment is developed between points *A* and *B*. This is eliminated by reversing the magnetic field and measuring the potential difference again between the points *A* and *B*.

8·10. THERMO-ELECTRIC EFFECTS :

With the help of free electron theory, the thermoelectric effects can be explained. Let us consider the base of two metals *A* and *B* having different electron density. Further let us suppose that the electron density in *A* is greater than electron density in metal *B*. Now the electronic pressure in *A* will be greater than in *B*. Due to the difference in electronic pressure, the electrons diffuse from *A* to *B*. This makes *A* positive and *B* negative. Thus a potential difference is created at the junction of two metals. When this potential difference reaches a certain value, it, prevents the migration of electrons from *A* to *B* and a state of equilibrium is set up. This explains that how a potential difference is created at the junction of two metals. We shall now apply this general conclusion to the two effects of thermo electric phenomena.

Peltier effect—In case of Peltier effect an external potential difference is applied to the junction *i.e.*, current is allowed to flow from *A* to *B*. Due to this current flow, there will be a transfer of electrons from *B* to *A*. As the electron density in *A* is greater than in *B*, hence certain amount of work is done against the electronic pressure difference. This involves the absorption of some energy at the junction which in consequence gets cooled. When the direction of the current is reversed, the electrons flow from *A* to *B* which make the energy available at junction in the form of heat *i.e.*, the junction gets heated. *The Peltier coefficient* π *is defined as the amount of energy liberated or absorbed when unit charge passes through the junction.* The expression for π can be derived as follows :